6) Calculate a speed of light, c, which would allow me to age two hours less than you while I fly to Rome and back next week. The distance between Roanoke and Rome is about 5000 miles and the plane flies at about 500 miles per hour.

$$t_{lab} = 2\,\frac{5000}{500} = 20 \text{ hrs}$$

$$t_{voy} = 18 \text{ hrs}$$

$$\gamma = \frac{20}{18} \qquad \beta = \sqrt{1 - \frac{1}{\gamma^2}} = \frac{V_{plane}}{c}$$

$$c = \frac{V_{plane}}{\sqrt{1 - \frac{1}{\gamma^2}}} = \frac{500}{\sqrt{1 - \left(\frac{18}{20}\right)^2}} = 1147 \text{ mph}$$

8) A photon of energy, E= 10 keV, scatters from an electron at rest. The outgoing photon is at 90 degrees from the incident photon. What kinetic energy does the recoil electron have?

$$\frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 + \cos\theta)$$

$$E' = \frac{1}{\frac{1}{m_e c^2} + \frac{1}{E}} = \frac{1}{\frac{1}{511} + \frac{1}{10}} = 9.81 \text{ keV}$$

$$KE = E - E' = 10 \text{ keV} - 9.81 \text{ keV} = 191 \text{ eV}$$

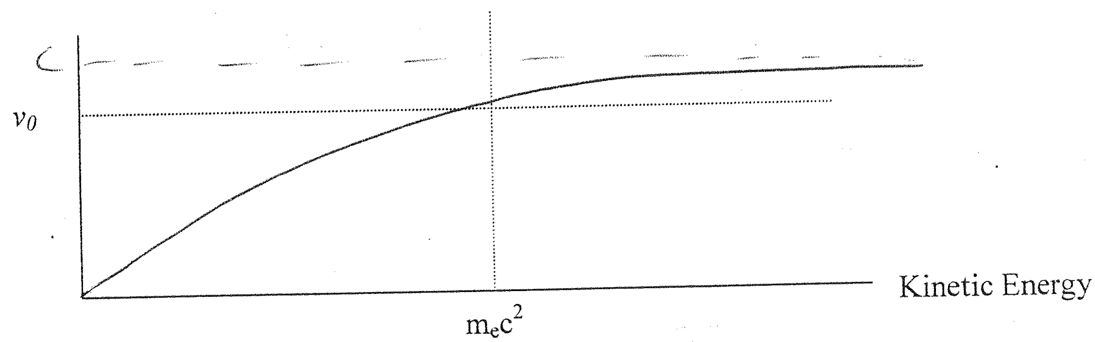
1, 2, 7

1) (4 pts) Solve for the speed, v_0, where the kinetic energy of an electron equals its rest mass energy. Sketch a graph of speed, v, versus kinetic energy, k, for the electron

$(\gamma - 1) mc^2 = mc^2 \Rightarrow \gamma = 2 = \frac{1}{\sqrt{1-\beta^2}}$

$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{\frac{3}{4}}$

$v_0 = \sqrt{\frac{3}{4}} c = 0.866 c = 2.6 \times 10^8$ m/s



2) (2 pts) As I travel towards a radio station, I observe its broadcast frequency to be increased such that $\frac{\nu'}{\nu} = \sqrt{\frac{1+\beta}{1-\beta}}$. I observe its wavelength to also be changed such that

$\frac{\lambda'}{\lambda} = \frac{\nu}{\nu'} = \sqrt{\frac{1-\beta}{1+\beta}}$

7) (6 pts) A Δ^+ particle decays from rest into a proton and pion. The pion has a rest mass energy of 140 MeV and momentum of 230 MeV/c. The proton has a rest mass energy of 938 MeV. What is the rest mass energy of the Δ^+?

$$E_\pi = \sqrt{(m_\pi c^2)^2 + p_\pi^2 c^2}$$

$$= \sqrt{140^2 + 230^2} = 269.3 \text{ MeV}$$

$$E_p = \sqrt{938^2 + 230^2} = 965.8 \text{ MeV}$$

$$m_\Delta c^2 = E_\pi + E_p = 1235 \text{ MeV}$$

1) Steve and Alma are the same age on Earth. Steve travels to a space station 10 light years away at 0.6 c. Alma decides to wait and leaves later on a faster ship which travels at 0.9 c. They both arrive at the same time. Who is older when they arrive and by how much?

On earth, Steve's time of travel is:

$$\frac{10}{0.6} = 16.7\ y$$

but since $\gamma = \frac{1}{\sqrt{1-0.6^2}} = 1.25$, Steve thought

the trip took only $\frac{16.7}{1.25} = 13.4$ years saving

his aging by $16.7 - 13.4 = 3.3\ y.$

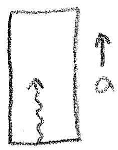
Repeat for Alma: $\frac{10}{0.9} = 11.1\ y$ $\quad \gamma = \frac{1}{\sqrt{1-(0.9)^2}} = 2.3$

$\frac{11.1}{2.3} = 4.85$ years $\therefore$ saves $11.1 - 4.8 = 6.3$

$\therefore$ Alma younger by $6.3 - 3.3 = 3.0\ y$

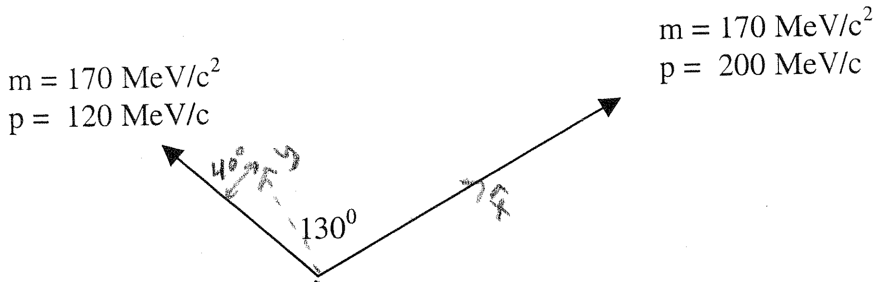
6) Describe a thought experiment which would indicate the necessity of a gravitational red-shift for light.

Consider light travelling from the floor to ceiling of an accelerating room.



By the time the light reaches the ceiling the detector is moving faster than the source so there is a doppler shift. However, since a gravitational field and an accelerating field can not be told apart, there must also be a doppler shift for light moving up or down in a gravitational field.

8) A particle *at rest* decays into three other particles. One of them is a photon and not observed. The other two have energies and momentum shown. What is the rest mass energy of the original particle?



$E_i = E_f$

$\vec{P}_i = \vec{P}_f = 0$

E_γ

$p_\gamma = \frac{E_\gamma}{c}$

$$E_1 = \sqrt{(mc^2)^2 + p^2c^2}$$

$$= \sqrt{170^2 + 120^2} = 208 \text{ MeV}$$

$$E_2 = \sqrt{170^2 + 200^2} = 263 \text{ MeV}$$

$$p_{\gamma_x} = 120 \sin 40 - 200 = -123 \text{ MeV/c}$$

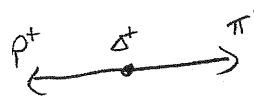
$$p_{\gamma_y} = -120 \cos 40 = -91.9 \text{ MeV/c}$$

$$\Rightarrow p_\gamma = \sqrt{123^2 + 91.9^2} = 153 \text{ MeV/c}$$

$$\Rightarrow E_\gamma = 153 \text{ MeV}$$

$$E_i = m_i c^2 = 208 + 263 + 153 = 624 \text{ MeV}$$

1) 1) A Δ^+ particle decays from rest into a proton and pion. The pion has a rest mass energy of 140 MeV and momentum of 230 MeV/c. The proton has a rest mass energy of 938 MeV. What is the rest mass energy of the Δ^+?



$$\vec{P}_i = \vec{P}_f = 0 \quad \Rightarrow \quad \vec{P}_p = -\vec{P}_{\pi^0}$$

$$E_i = E_f$$

$$\sqrt{m_{\Delta^+}^2 c^4 + \cancel{P_{\Delta^+}^2 c^2}^{\,0}} = E_p + E_{\pi^0} = M_{\Delta^+} c^2$$

$$E_p = \sqrt{P_p^2 c^2 + m_p^2 c^4} = \sqrt{230^2 + 938^2} = 966 \text{ MeV}$$

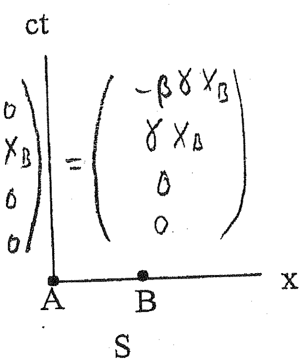
$$E_{\pi^0} = \sqrt{230^2 + 140^2} = 269 \text{ MeV}$$

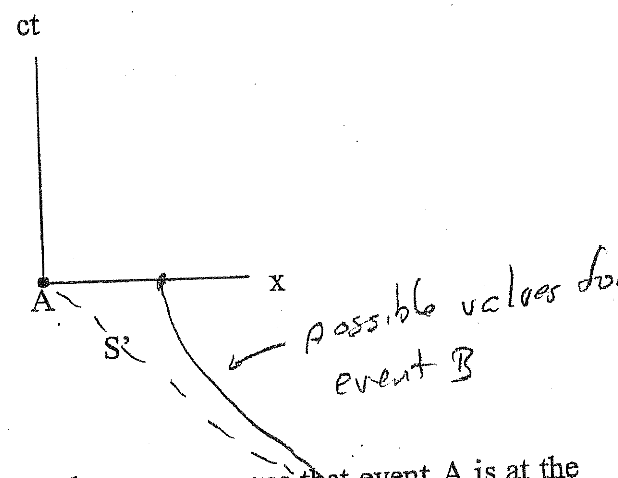
$$m_{\Delta^+} c^2 = 966 + 269 = 1235 \text{ MeV}$$

2) a) Two events, A and B, happen in frame S at the same time as shown:

for event B:

$$\begin{pmatrix} c\bar t\\ \bar x\\ \bar y\\ \bar z\end{pmatrix} = \begin{pmatrix}\gamma & -\beta\gamma & & \\ -\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1\end{pmatrix}\begin{pmatrix}0\\ x_B\\ 0\\ 0\end{pmatrix} = \begin{pmatrix}-\beta\gamma x_B\\ \gamma x_B\\ 0\\ 0\end{pmatrix}$$

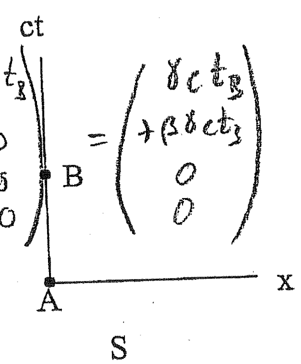


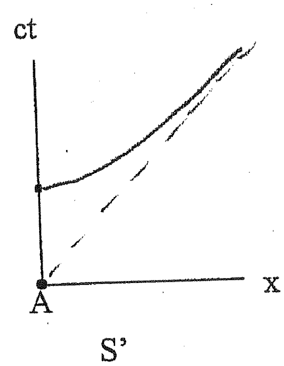


In another frame, S', traveling to the *right* at speed v, one arranges that event A is at the origin. Sketch the locus of points where B could appear for different values of v.

b) Two events, A and B, happen in frame S at the same position as shown:

$$\begin{pmatrix} c\bar t\\ \bar x\\ \bar y\\ \bar z\end{pmatrix} = \begin{pmatrix}\gamma & +\beta\gamma & & \\ +\beta\gamma & \gamma & & \\ & & 1 & \\ & & & 1\end{pmatrix}\begin{pmatrix}ct_B\\ 0\\ 0\\ 0\end{pmatrix} = \begin{pmatrix}\gamma c t_B\\ +\beta\gamma c t_B\\ 0\\ 0\end{pmatrix}$$

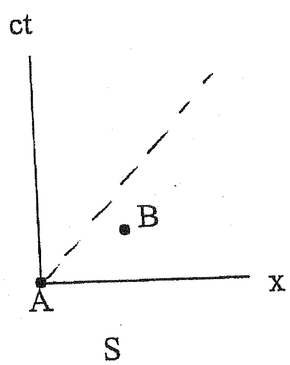




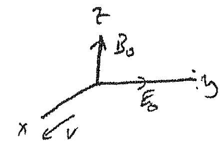
3

In another frame, S', traveling to the *left* at speed v, one arranges that event A is at the origin. Sketch the locus of points where B could appear for different values of v.

c) Two events, A and B, occur in frame S as shown (both axes have the same scale). Could they be causally related? Why?



below light cone ∴ can not be causally related.



3) Given $A^\mu = \begin{pmatrix} -yE_o/c \\ -yB_o \\ 0 \\ 0 \end{pmatrix}$ in frame S, calculate the magnetic and electric fields $\vec{E}$ and $\vec{B}$, where E_0 and B_0 are constants.

$$A^\mu = \begin{pmatrix} \phi/c \\ A_x \\ A_y \\ A_z \end{pmatrix} \Rightarrow \phi = -yE_0 \qquad \vec{A} = -yB_0\,\hat{x}$$

$$\vec{E} = -\left(\nabla\phi + \frac{\partial \vec{A}}{\partial t}\right) = E_0\hat{y}$$

$$\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ -yB_0 & 0 & 0 \end{vmatrix} = B_0\hat{z}$$

Now boost A into a new frame S' moving in the x-direction at speed v.

$$\bar{A}^\mu = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -yE_0/c \\ -yB_0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -y\gamma E_0/c + \beta\gamma y B_0 \\ \beta\gamma y E_0/c - \gamma y B_0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\gamma\bar{y}}{c}(-E_0 + vB_0) \\ \gamma\bar{y}\left(-B_0 + \frac{vE_0}{c^2}\right) \\ 0 \\ 0 \end{pmatrix}$$

note $\bar{y} = y$

In frame S', calculate $\vec{E}$' and $\vec{B}$' from this new A'.

$$\vec{E} = -\left(\nabla\phi + \frac{\partial A}{\partial t}\right)_{\text{wrt new coords}} = \gamma(E_0 - vB_0)\hat{y}$$

$$\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ \gamma y\left(-B_0 + \frac{vE_0}{c^2}\right) & 0 & 0 \end{vmatrix} = \gamma\left(B_0 - \frac{vE_0}{c^2}\right)\hat{z}$$

wrt new coords.

Show that these agree with the anticipated transformations for E and B fields derived in class.

$$\vec{E}' = E_{\parallel} + \gamma(E_\perp + \vec{v}\times\vec{B}) = \gamma(E_0\hat{y} + -vB_0\hat{y}) \quad ✓$$

$$\vec{B}' = B_{\parallel} + \gamma\left(B_\perp - \frac{\vec{v}\times\vec{E}}{c^2}\right) = \gamma\left(B_0\hat{z} - \frac{vE_0}{c^2}\hat{z}\right) \quad ✓$$

... a little bit fishy, but works in this

$$A^\mu = \begin{pmatrix} -yE/c \\ 0 \\ xB \\ 0 \end{pmatrix} \qquad \vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t} = -\nabla(-yE) = E\hat{y}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & xB & 0 \end{vmatrix} = B\hat{z}$$

$$\bar{A}^\mu = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -yE/c \\ 0 \\ xB \\ 0 \end{pmatrix} = \begin{pmatrix} -y\gamma E/c \\ y\beta\gamma E/c \\ xB \\ 0 \end{pmatrix}$$

but $\bar{x} = \Lambda x \quad x = \Lambda^{-1}\bar{x}$

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & +\beta\gamma & 0 & 0 \\ +\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c\bar{t} \\ \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix} \Rightarrow \begin{aligned} x &= +\beta\gamma c\bar{t} + \gamma\bar{x} \\ y &= \bar{y} \\ z &= \bar{z} \end{aligned}$$

so $\bar{A}^\mu = \begin{pmatrix} -\bar{y}\gamma E/c \\ \bar{y}\beta\gamma E/c \\ \gamma(\bar{x} + v\bar{t})B \\ 0 \end{pmatrix}$

so in new frame take derivatives wrt new coords

$$\bar{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t} = \gamma E\hat{y} + \gamma vB\hat{y} = \gamma(E - vB)\hat{y}$$

$$\bar{B} = \bar{\nabla}\times\bar{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial \bar{x}} & \frac{\partial}{\partial \bar{y}} & \frac{\partial}{\partial \bar{z}} \\ \bar{y}\beta\gamma E/c & \gamma(\bar{x}+v\bar{t})B & 0 \end{vmatrix} = (\gamma B + \beta\gamma E/c)\hat{y} = \gamma\left(B + \frac{v}{c^2}E\right)\hat{y}$$

4) Suppose you have a collection of particles, all moving in the x direction, with total energies E_1, E_2, E_3 and momenta p_1, p_2, p_3. Find the velocity of the ***center of momentum*** frame, in which the total momentum is zero. (Hint: transform the total momentum into the new frame)

$$E_T = E_1 + E_2 + E_3$$

$$P_T = P_1 + P_2 + P_3 \qquad \text{(all in x-direction)}$$

$$\begin{pmatrix} \bar{E}_T/c \\ \bar{P} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E_T/c \\ P_T \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \gamma E_T/c - \beta\gamma P_T \\ -\beta\gamma E_T/c + \gamma P_T \\ 0 \\ 0 \end{pmatrix}$$

but we want $\bar{P} = 0$

$$\Rightarrow \quad 0 = \gamma\left(-\beta E_T/c + P_T\right)$$

$$\Rightarrow \quad \beta E_T/c = P_T$$

$$V = c^2 \frac{P_T}{E_T} = c^2 \frac{P_1 + P_2 + P_3}{E_1 + E_2 + E_3}$$

5) In class we showed that $\frac{dE}{dt} = \vec{F} \cdot \vec{u}$ while deriving the Work-Energy theorem. Re-derive this, in a much simpler fashion, by taking the time derivative of $p^{\mu} p_{\mu} = -m^2 c^2$, showing each step clearly.

$$\frac{d}{dt}\left(p^{\mu} p_{\mu} = \frac{-E^2}{c^2} + \vec{p}^{\,2} = -m^2 c^2 \right)$$

$$\frac{-2E}{c^2} \frac{dE}{dt} + 2\,\vec{p} \cdot \frac{d\vec{p}}{dt} = 0$$

but: $\frac{d\vec{p}}{dt} = \vec{F}$

$\vec{p} = \gamma m \vec{u}$

$E = \gamma m c^2$

$$\Rightarrow \frac{-2\gamma m c^2}{c^2} \frac{dE}{dt} + 2\gamma m \vec{u} \cdot \vec{F} = 0$$

$$\Rightarrow \frac{dE}{dt} = \vec{u} \cdot \vec{F}$$

6) a) Consider the equation of a traveling plane wave moving along a direction $\vec{k}$. What four-vector is $\vec{k}$ the spatial part of?

$\psi \propto e^{i(\vec{k}\cdot\vec{x} - \omega t)}$

$$k^{\mu} = \begin{pmatrix} \omega/c \\ k_x \\ k_y \\ k_z \end{pmatrix}$$

but $x^{\mu} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$

2

Interpret the Lorentz invariant contraction $k^{\mu}x_{\mu}$:

"phase" same for all observers

1

What is $k^{\mu}k_{\mu}$ for light in vacuum?

$K^{\mu}K_{\mu} = \frac{-\omega^2}{c^2} + \vec{K}^2$ but for light $v = f\lambda = c = \frac{\omega}{K}$

$= k^2 - k^2$

$= 0$

2

b) Is angular momentum, $\vec{L}$, the spatial part of a four-vector? Explain

$\vec{L} = \vec{r} \times \vec{p}$

$L_i = r_j p_k - r_k p_j \Rightarrow L_{\mu} = \epsilon_{\mu\nu\delta}\, r_{\nu} p_{\delta}$

made from 2 4-vectors = transforms like second-rank tensor

i.e. not a 4-vector

1) A clock is at rest in the train station. You are on the train passing by at 0.6c. How long does each second on the clock appear to take to you?

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.6^2}} = \frac{1}{.8} = 1.25$$

$$t = \gamma t_0$$

$\boxed{1.25\,s}$

2) An object of volume V in its rest frame passes you at 0.6c. What is its apparent volume to you?

$$dV = dx\,dy\,dz$$

$$\left.\begin{aligned} dx' &= dx \\ dy' &= dy \\ dz' &= \frac{dz}{\gamma} \end{aligned}\right\} dV' = \frac{dx\,dy\,dz}{\gamma}$$

$$V' = \frac{V}{\gamma} = 0.8\,V$$

$\boxed{0.8\,V}$

3) A photon knocks an electron from a surface with a kinetic energy of 2 eV. The work function of the surface is 1 eV. What is the minimum wavelength of the photon?

$$hf = \Phi + KE = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Phi + KE} = \frac{2\pi\hbar c}{\Phi + KE} = \frac{2\pi\, 197\ \text{eV nm}}{(2+1)\,\text{eV}} =$$

$\boxed{413\,\text{nm}}$

4) Estimate the density of nuclear matter (give result in g/cm^3):

$\boxed{\sim 2\times10^{14}\ g/cm^3}$

$$\rho = \frac{m}{V} = \frac{938\ \text{MeV}/c^2\ \ 1.6\times10^{-13}\ \text{J/MeV}}{\frac{4}{3}\pi\,(1.2\times10^{-15})^3} = 2.3\times10^{17}\ \frac{kg}{m^3} = 2.3\times10^{14}\ g/cm^3$$

10) An electron is accelerated from rest by a uniform electric field of 100 kV/cm. After it has traveled 5 cm, what is its speed?

$KE = 5(100) = 500 \text{ keV}$

$mc^2 = 511 \text{ keV}$

$E = \gamma mc^2 \qquad \gamma = \frac{E}{mc^2} = \frac{1.011}{0.511} = 1.98$

but $\gamma = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.863$

$\Rightarrow v = 0.86\,c = 2.6 \times 10^8 \text{ m/s}$

1) An electron is accelerated from rest through a 5.00 MeV potential, and then travels through a beam-line 100 m long in the laboratory frame.

$KE = 5.00 \text{ MeV} = (\gamma - 1) mc^2 = (\gamma - 1)\, 0.511$

$\Rightarrow \gamma = 10.8 = \dfrac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \qquad \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = 0.996$

a) How long does the electron take to pass through the beamline in the lab frame?

$t = \dfrac{100}{.996 \times 3 \times 10^8} = 0.335 \ \mu s$

b) How long does the electron take to pass through the beamline in the electron's frame?

$t_0 = \dfrac{t}{\gamma} = 0.0310 \ \mu s$

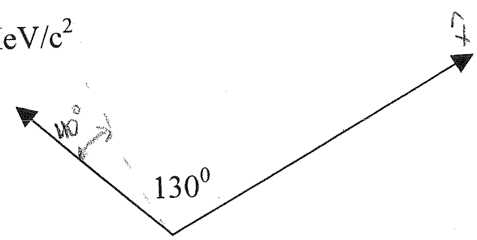
c) How long is the beam-line in the electron's frame?

$l = \dfrac{l_0}{\gamma} = \dfrac{100}{10.8} = 9.3 \text{ m}$

10) A moving particle decays into two other particles as shown. What is the rest mass energy of the original particle?

m = 170 MeV/c^2
v = 0.90 c

m = 170 MeV/c^2
v = 0.80 c



$\gamma = \frac{1}{\sqrt{1-\beta^2}}$

$= 1.667$

$p = \gamma m v = 226.7$ MeV/c

$\gamma = 2.294$

$p = 351.0$ MeV/c

$E_i = E_f = 1.667(170) + 2.294(170)$

$= \sqrt{170^2 + (226.7)^2} + \sqrt{170^2 + 351^2} = 673.4$ MeV

$p_i = p_f$

$p_x = 351 - 227 \sin 40 = 205.1$ MeV/c

$p_y = 227 \cos 40 = 173.9$ MeV/c

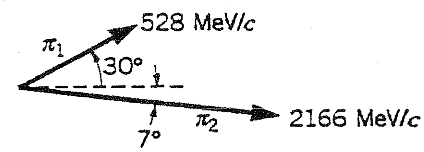
$p = \sqrt{p_x^2 + p_y^2} = 268.9$ MeV/c

so: $mc^2 = \sqrt{E_i^2 - p_i^2 c_i^2}$

$= \sqrt{(673.4)^2 - (269)^2}$

$= 617.4$ MeV

2) Consider a particle traveling along the x-axis to the right which decays into two pions with momentum as shown. The mass of the pions is about 140 MeV/c^2. We will determine the mass of the parent ρ meson.



a) What is the *total* energy of π_1 and π_2 combined? (Calculate them separately first.)

$$E_1 = \sqrt{p^2c^2 + (m_\pi c^2)^2} = \sqrt{528^2 + 140^2} = 546 \text{ MeV}$$

$$E_2 = \quad '' \quad \sqrt{2166^2 + 140^2} = 2171 \text{ MeV}$$

$$E_T = E_1 + E_2 = 2717 \text{ MeV}$$

b) What is the total momentum in the x and y directions?

$$P_x = 528 \cos 30 + 2166 \cos 7 = 2607 \text{ MeV/c}$$

$$P_y = 528 \sin 30 - 2166 \sin 7 = 0$$

c) Using conservation of energy and momentum, what is the total momentum and energy of the parent particle?

$$E_T = 2717 \text{ MeV}$$

$$\vec{p} = 2607 \text{ MeV/c } \hat{x} + 0 \ \hat{y}$$

d) What is the rest mass energy of the parent particle?

$$m_T c^2 = \sqrt{E^2 - p^2c^2} = 765 \text{ MeV}$$

e) After many such measurements, one accumulates the figure below. Using it, estimate the lifetime of the parent ρ meson.

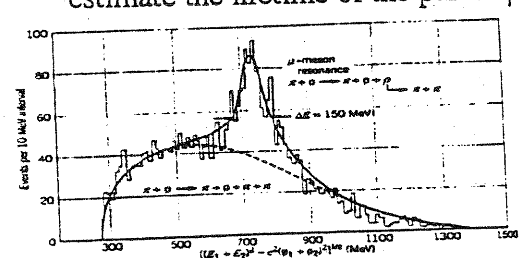


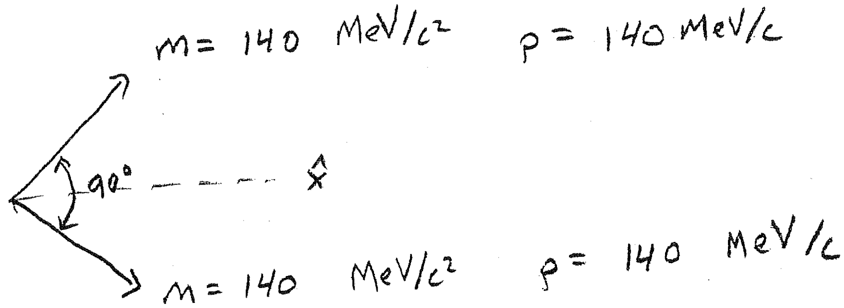
FIGURE 14.4 The resonance identified as the ρ meson. The horizontal axis shows the energy and momenta of the two decay pi mesons, combined to be equivalent to the mass of the resonance particle.

$$\Delta E \Delta t = \hbar$$

$$\Delta t = \frac{\hbar}{\Delta E} = \frac{6.58 \times 10^{-22} \text{ MeV s}}{150 \text{ MeV}}$$

$$= 4 \times 10^{-24} \text{ s}$$

1) A neutral particle decays into two charged ones with characteristics as shown. How fast was the neutral particle traveling?



$E_1 = \sqrt{(mc^2)^2 + p^2c^2} = \sqrt{140^2 + 140^2} = \sqrt{2}\ 140$ MeV

$E_2 =$ 〃

$E_T = 2\sqrt{2}\ 140$ MeV

$p_y = 0$

$p_x = 2 p_1 \cos 45 = 2\ 140\ \frac{\sqrt{2}}{2}$ MeV/c $= \sqrt{2}\ 140$ MeV/c

E conserved, p conserved

$$\beta = \frac{\gamma m v c}{\gamma m c^2} = \frac{pc}{E} = \frac{\sqrt{2}\ 140}{2\sqrt{2}\ 140} = \frac{1}{2}$$

1) If a typical 70-kg person were converted to energy and beamed to another location in a matter of 3 seconds, like in Star Trek, how much power would be in this beam?

$$P = \frac{E}{t} = \frac{mc^2}{t} = \frac{70\,(3\times10^8)^2}{3} = 2.1\times10^{18}\ W$$

2) Describe the properties of our universe which give rise to conservation of linear momentum, angular momentum, and energy.

rotational invariance → angular momentum conservation

translational " → linear " "

time invariance → energy conservation

3) An electron is accelerated from rest by a uniform electric field of 100 kV/cm. After it has traveled 5 cm, what is its speed?

$$KE = q\,\Delta V = 100\,\frac{kV}{cm}\,5\,cm\;e = 500\ keV$$

$$= (\gamma-1)mc^2$$

$$= (\gamma-1)\,511\ keV \Rightarrow \gamma = 1.98$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$1-\beta^2 = \frac{1}{\gamma^2}$$

$$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{4}} = 0.86 \Rightarrow v = 2.6\times10^8\ \frac{m}{s}$$

1) A certain particle has a total energy of 1.5 GeV, and a momentum given by $\vec{p} = \begin{bmatrix} 300 \\ 300 \\ 0 \end{bmatrix}$ MeV/c in the lab frame. Find the speed and direction of the frame (relative the lab frame) in which the particle has the least total energy, and what is that energy?

E_{min} when $\vec{p} = 0$

Speed in lab frame: $\beta = \frac{pc}{E} = \frac{\sqrt{0.3^2 + 0.3^2}}{1.5}$

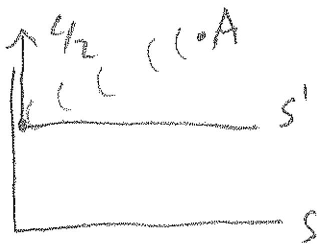
$= 0.283$

so, frame w/ $\vec{p} = 0$ moves at $0.283\,c$ ($= 8.5 \times 10^7$ m/s) at $45°$ above x-axis where

$E = mc^2 = \text{L.I.} = \sqrt{E^2 - p^2c^2} = \sqrt{1.5^2 - 0.3^2 - 0.3^2}$

$= 1.44$ GeV

2) Frames S and S' coincide at x=y=z=t=x'=y'=t'=0. Event A occurs at t=2 seconds and position (x,y,z)=($3\text{x}10^8$m, $5\text{x}10^8$m, 0) in the S frame. At what time in the S' frame will the light from this event pass the S' origin if the S' frame is moving along the y axis at the speed c/2?



$$A = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6E8 \\ 3E8 \\ 5E8 \\ 0 \end{pmatrix}$$

$\beta = 0.5$

$\gamma = \dfrac{1}{\sqrt{1-\beta^2}} = 1.155$

$$A' = \begin{pmatrix} 1.155 & 0 & -0.5774 & 0 \\ 0 & 1 & 0 & 0 \\ -1.5774 & 0 & 1.155 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 6E8 \\ 3E8 \\ 5E8 \\ 0 \end{pmatrix} = \begin{pmatrix} 4.04E8 \\ 3.0E8 \\ 2.311E8 \\ 0 \end{pmatrix}$$

so $t'_A = \dfrac{4.04E8}{3E8} = 1.35$ s

$(\text{dist})'$ to origin $= \sqrt{(3E8)^2 + (2.311E8)^2} = 3.787E8$

t' @ origin $= 1.35 + \dfrac{3.787E8}{3E8} = 2.61$ s

3) The neutrino beam from Fermilab to Soudan is produced by having energetic pions decay in-flight (producing neutrinos) along a 1200m tunnel. What is the maximum total energy of the pion if you want half of those entering one end of the tunnel to decay before reaching the other end of the tunnel? (The rest mass of the pion is 135 MeV/c^2, and its proper half-life is 26 nanoseconds).

at speed c in lab frame pions only travel $26\times10^{-9}\ 3\times10^{8} = 7.8$ m before $\frac{1}{2}$ decay w/o time dilation, so $\gamma = \frac{1200}{7.8} = 154$

so $E = \gamma mc^2 = 154(135) = 20.8$ GeV

$(= 3.3\times10^{-9}\ J)$

– or –

$\vec{\eta} = \frac{d\vec{x}}{d\tau} = \frac{1200\ m}{26\times10^{-9}\ s} = 4.6\times10^{10}\ m/s = 154c$

$\vec{p} = m\vec{\eta}$

$E = \sqrt{(mc^2)^2 + p^2c^2}$

$= \sqrt{(135)^2 + [135(154)]^2}$

$= 20.8$ GeV

(recall $\eta^\nu\eta_\nu = -c^2\gamma^2 + \vec{\eta}^2 = L.I. = -c^2$

ie: $|\vec{\eta}|$ can be $> c$!!)

4) An unknown neutral particle (invisible in the detector) decays symmetrically in flight into two singly charged particles (opening angle of 60 degrees), each with mass 450 MeV/c^2 and radius of curvature of 0.75 m in a 3 Tesla magnetic field as shown. What was the mass of the unknown particle? (hint: this requires a little playing with units – if everything goes in as MKS, then the result is in MKS; it *may* be convenient to multiply by "1" in the form of $(3.0\text{x}10^8$ m/s) / c) If you can not get the momentum of the particles, then use 500 MeV/c each.

$$p = Brq$$

$$= \left(3 \;\; 0.75 \;\; 1.6\times10^{-19} \;\; kg \frac{m}{s}\right)\left(\frac{3\times10^{8}\frac{m}{s}}{c} \;\; \frac{eV}{1.6\times10^{-19} J}\right)$$

$$= 675 \text{ MeV/c}$$

$$\Rightarrow E = \sqrt{p^2c^2 + m^2c^4} = \sqrt{675^2 + 450^2} = 811 \text{ MeV}$$

$$E_i = E_f = 1622 \text{ MeV}$$

$$P_{i_x} = P_{f_x} = 2(675)\cos 30 = 1169 \text{ MeV/c}$$

$$P_{i_y} = P_{f_y} = 0$$

$$\Rightarrow m = \sqrt{E^2 - p^2c^2}/c^2 = \sqrt{1622^2 - 1169^2} = 1124 \text{ MeV/c}^2$$

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if used $p = 500$ MeV/c

$$E = 673 \text{ MeV} \Rightarrow E_i = 1345 \text{ MeV}$$

$$p_i = 2(500)\cos 30 = 866 \text{ MeV/c}$$

$$\Rightarrow m = \frac{\sqrt{1345^2 - 866^2}}{c^2} = 1029 \text{ MeV/c}^2$$

5) A deuterium particle (assumed twice the mass of a proton) traveling at 0.4c collides head-on with a proton traveling in the opposite direction. What energy must the proton have, such that the total momentum of the two particles in the laboratory frame be zero?

$m = 2(938\ \text{MeV}/c^2)$ $\qquad$ $m = 938\ \text{MeV}/c^2$

$V = 0.4c$

$$p = \gamma m v$$

$$= \gamma m c^2 \beta / c$$

$$= \frac{1}{\sqrt{1-(0.4)^2}}\, 2(938)\,\frac{0.4}{c} = 818.8\ \text{MeV}/c$$

$$P = 0 = 818.8 - P_p \quad \Rightarrow P_p = 818.8\ \text{MeV}/c$$

$$E_p = \sqrt{938^2 + 819^2} = 1245\ \text{MeV}$$

$$E_k = 1245 - 938 = 307\ \text{MeV}$$

6) You see a red light ahead of you but don't want to have to stop. So you speed up until the light appears green. How fast are you going? (Red light has a wavelength of 650 nm, and green 510 nm.)

$$\frac{f_{obs}}{f_{source}} = \sqrt{\frac{1+\beta}{1-\beta}} = \frac{\lambda_s}{\lambda_o}$$

$$\left(\frac{\lambda_s}{\lambda_o}\right)^2 = \frac{1+\beta}{1-\beta}$$

$$\left(\frac{\lambda_s}{\lambda_o}\right)^2 - \frac{\lambda_s^2}{\lambda_o^2}\beta = 1+\beta$$

$$\left(\frac{\lambda_s}{\lambda_o}\right)^2 - 1 = \left(1 + \frac{\lambda_s^2}{\lambda_o^2}\right)\beta$$

$$\beta = \frac{\left(\frac{\lambda_s}{\lambda_o}\right)^2 - 1}{\left(\frac{\lambda_s}{\lambda_o}\right)^2 + 1} = \frac{\left(\frac{650}{510}\right)^2 - 1}{\left(\frac{650}{510}\right)^2 + 1} = 0.238$$

$$\Rightarrow v = 0.238c = 7.1 \times 10^7 \text{ m/s}$$

7) John gets on a space ship traveling away from Earth at 0.5c. After a year, his wife starts to miss him, and sends him a radio signal asking him to come back. When John receives the message, he dutifully turns around and heads back at 0.5c. How much younger will John be than his wife when he gets home?

In wife's frame, John receives message

when $0.5ct = 1.0c(t-1y) \Rightarrow t = 2$ years

$\Rightarrow$ John gone total of 4 years

but $\gamma = \frac{1}{\sqrt{1-(0.5)^2}} = 1.155$

$\Rightarrow$ John only ages $\frac{4}{1.155} = 3.46$ years

$\Rightarrow$ John 0.54 years younger than wife

8) A 1.0 MeV photon collides with an electron. What is the maximum *kinetic* energy which can be given to the electron? **Solve this directly using energy and momentum conservation**, and then check you results using Eqn 3-40 for Compton scattering: $\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta)$ (note: hc=1240 eVnm)

$$E: \quad E_\gamma + mc^2 = E_\gamma' + \sqrt{(mc^2)^2 + p_e^2 c^2}$$

$$p: \quad \frac{E_\gamma}{c} = p_e - \frac{E_\gamma'}{c} \Rightarrow p_e = (E_\gamma + E_\gamma')/c$$

$$\Rightarrow (E_\gamma - E_\gamma' + mc^2)^2 = (mc^2)^2 + (E_\gamma + E_\gamma')^2$$

$$\cancel{E_\gamma^2} - 2E_\gamma E_\gamma' + \cancel{E_\gamma'^2} + 2(E_\gamma - E_\gamma')mc^2 + \cancel{m^2c^4} = \cancel{m^2c^4} + \cancel{E_\gamma^2} + 2E_\gamma E_\gamma' + \cancel{E_\gamma'^2}$$

$$2(E_\gamma - E_\gamma') = \frac{4E_\gamma E_\gamma'}{mc^2}$$

$$\frac{1}{E_\gamma'} - \frac{1}{E_\gamma} = \frac{2}{mc^2}$$

$$\Rightarrow E' = \frac{1}{\frac{1}{E_\gamma} + \frac{2}{mc^2}} = \frac{1}{1 + \frac{2}{0.511}} = 0.204 \text{ MeV}$$

$$\Rightarrow E_{e_K} = 1.0 - 0.204 = 0.80 \text{ MeV}$$